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**LETTER TO THE EDITOR**

# 'True' self-avoiding Lévy flights

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**Abstract.** We consider 'true' self-avoiding Lévy flights, defined by a probability decreasing like $r^{-\sigma}$ of making a step of length larger than $r$, with a tendency to avoid already visited sites. The predictions of a Flory argument for the upper critical dimension and for the exponent of the gyration radius appear to be in disagreement with those of a renormalised field theory.

The equilibrium statistics of self-avoiding Lévy flights, where the probability of a step being of greater length than some $r$ decreases like $r^{-\sigma}$ $(0<\sigma<2)$, has been recently considered by Grassberger (1985). While this problem is purely static, it may be simulated by a process much in the same way as the equilibrium statistics of a self-repelling chain is simulated by a self-avoiding walk, where the walker is suppressed if it tries to retrace its steps.

A variation on the theme of self-avoiding walks is the 'true' self-avoiding walk (Amit *et al* 1983). In this model, if a walker attempts to retrace its steps, instead of being suppressed, is deviated to sites not yet (or less often) visited. We have considered this variation on a Lévy flight by means both of a Flory argument and of field theoretical considerations. While for all other instances both lines of argument agree in the location of the upper critical dimension $d_c$—and the Flory argument appears as a reliable guide for the value of the gyration radius exponent $\nu$ below $d_c$—we find a disagreement in their predictions for this model.

We obtain in fact

(i) from the Flory argument:

$$d_c=\sigma; \qquad \begin{array}{ll} \nu=1/\sigma & d>d_c \\ \nu=2(d+\sigma) & d<d_c; \end{array} \tag{1}$$

(ii) from field theory:

$$d_c=2(\sigma-1), \qquad \nu=1/\sigma \qquad \forall d. \tag{2}$$

The prediction of field theory is that the asymptotic behaviour of the flight for $d$ less than $d_c$ is different from that of a non-interacting Lévy flight, although the critical exponent $\nu$ does not change.

We now expound the arguments leading to equations (1) and (2). Equation (1) derives from a Flory argument adapted to kinetic processes (Family and Daoud 1984, Family 1984). One has a repelling potential proportional to $N/R^d$ instead of $N^2/R^d$ as in the self-avoiding Lévy flight case. The entropy term is taken to be $R^\sigma/N$

(Grassberger 1985). Minimising the free energy with respect to $R$ we obtain $R \sim N^{\nu}$ where

$$\nu_F = 2/(d+\sigma). \tag{3}$$

The upper critical dimension is identified by the condition that $\nu$ is equal to its 'free' value $1/\sigma$.

The field theoretical formulation follows from standard techniques (Grassberger and Scheunert 1980). One obtains the action:

$$S = \int \mathrm{d}t \int \mathrm{d}^d r \left[ \tilde{\psi}\left( -\frac{\partial \psi}{\partial t} + D\nabla^{\sigma}\psi \right) - g\tilde{\psi}\nabla\psi \cdot \nabla \int_0^t \mathrm{d}t' \tilde{\psi}(t')\psi(t') \right]. \tag{4}$$

One easily sees from power counting that the dimension of the coupling constant $g$ is given by $2(\sigma-1)-d$, which identifies the upper critical dimension given by (2). On the other hand one finds no diagrams renormalising the $\tilde{\psi}\nabla^{\sigma}\psi$ term in the free Lagrangian (a similar property also holds in the equilibrium case: see e.g. Fisher *et al* 1972). We thus obtain, exactly to all orders in perturbation theory:

$$2 - \eta = \sigma. \tag{5}$$

From the fact that $\gamma = 1$, due to probability conservation, and from the scaling relation $\gamma/\nu = 2 - \eta$, the estimation in (2) follows. It is possible that one is witnessing a breakdown of scaling due to the presence of a 'dangerous' irrelevant operator.

One can also consider 'true' self-avoiding Lévy flights with long-range repelling potentials (Peliti and Zhang 1985, Zhang 1985). The potential is then taken to be:

$$\phi(\boldsymbol{r}) = \int \mathrm{d}^d r' \frac{\rho(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^{\omega}}. \tag{6}$$

A similar Flory argument yields

$$\nu = 2/(\omega+\sigma) \qquad \text{for } \omega < \sigma. \tag{7}$$

In particular, for repelling Coulomb interactions ($\omega = d-2$) we obtain

$$\nu = 2/(d+\sigma-2), \tag{8}$$

which gives in turn

$$d_c = \sigma + 2. \tag{9}$$

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